Reliably distinguishing states and Quantum algorithm proofs

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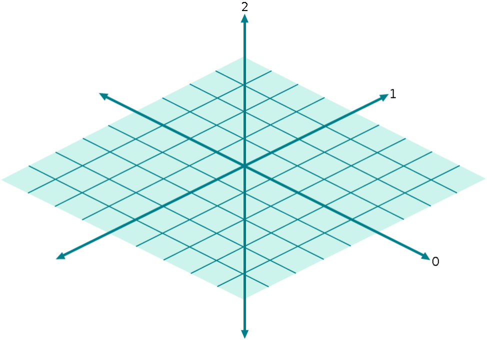
Reliably distinguishing qutrit states using one-way LOCC

What is a qutrit?

A **qutrit** is a three-level quantum unit of information.

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle$$



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$$
\begin{aligned}
|\psi\rangle &= \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle \\
&= \alpha_0 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}
\end{aligned}
$$

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$$\begin{aligned}|\psi\rangle &= \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle \\ &= \alpha_0\begin{pmatrix}1\\0\\0\end{pmatrix} + \alpha_1\begin{pmatrix}0\\1\\0\end{pmatrix} + \alpha_2\begin{pmatrix}0\\0\\1\end{pmatrix}\end{aligned}$$

$$\begin{aligned}P|0\rangle &= |\alpha_0|^2 \\ P|1\rangle &= |\alpha_1|^2 \\ P|2\rangle &= |\alpha_2|^2\end{aligned}$$

System and environment

$$U(|\psi\rangle\langle\psi| \otimes |\varepsilon\rangle\langle\varepsilon|)U^\dagger$$

where

- $|\psi\rangle\langle\psi|$ is the system,
- $|\varepsilon\rangle\langle\varepsilon|$ is the environment,
- U describes the noise due to or interaction with the environment.

Product Measurements

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For a product measurement on $|\varphi\rangle$, we choose one basis for the first system, then second basis for second system.

Entangled Measurements

Project on $2n$ *entangled* orthonormal vectors in $\mathbb{C}^2 \otimes \mathbb{C}^n$ as basis.
Example: four vectors for $\mathbb{C}^2 \otimes \mathbb{C}^2$.

$$|e_1\rangle, \quad |e_2\rangle,$$
$$|e_3\rangle, \quad |e_4\rangle.$$

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Given 3 orthonormal states in $\mathbb{C}^3 \otimes \mathbb{C}^n$, can we distinguish with certainty using a product measurement?

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Hint

It is always possible to use product measurements in $\mathbb{C}^2 \otimes \mathbb{C}^n$.

Answer

Conjecture

Yes!

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We show that the order of these measurements is not arbitrary as in $\mathbb{C}^2 \otimes \mathbb{C}^n$.

Optimization as Technique

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  - Search over rotations of these vectors
  - Maximize mutual orthogonality of states

Open Problems

- Analytical proof of conjecture

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- Analytical proof of conjecture
- “Ping-pong” protocol for higher dimensions

Further Reading

- **Reliably distinguishing states in qutrit channels using one-way LOCC**
  Christopher King and Daniel Matysiak
  Preprint: `http://arxiv.org/abs/quant-ph/`**0510004**

Quantum algorithm semantics and proof systems

What is QPL?

Quantum Programming Language proposed by Selinger.

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Quantum Programming Language proposed by Selinger.

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- clear semantics
- classical control, quantum data
- flowchart (program) is a linear map from vector space to vector space
- superoperators

Goal

The main goal is to develop proof systems to prove properties of quantum algorithms.

The End