Reliably distinguishing states in qutrit channels using one-way LOCC

Daniel Matysiak
dmatysiak@ccs.neu.edu
Northeastern University

November 10, 2005

Prelude

What is and Why Quantum Computing and Information?

What is and Why Quantum Computing and Information?

What?

What is and Why Quantum Computing and Information?

What?

- harnessing of quantum mechanics for information processing

What is and Why Quantum Computing and Information?

What?

- harnessing of quantum mechanics for information processing

Why?

What is and Why Quantum Computing and Information?

What?

- harnessing of quantum mechanics for information processing

Why?

- information and computation are **physical**

What is and Why Quantum Computing and Information?

What?

- harnessing of quantum mechanics for information processing

Why?

- information and computation are **physical**
- miniaturization of transistors

What is and Why Quantum Computing and Information?

What?

- harnessing of quantum mechanics for information processing

Why?

- information and computation are **physical**
- miniaturization of transistors
- do things impossible in *classical* computing

What is and Why Quantum Computing and Information?

What?

- harnessing of quantum mechanics for information processing

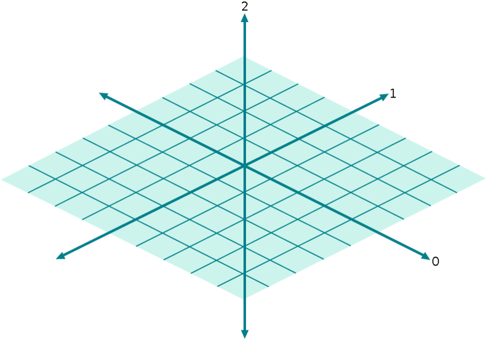
Why?

- information and computation are **physical**
- miniaturization of transistors
- do things impossible in *classical* computing
- to better understand decoherence

What is a qutrit?

A **qutrit** is a three-level quantum unit of information.

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle$$



What is a qutrit?

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle$$

What is a qutrit?

$$\begin{aligned} |\psi\rangle &= \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle \\ &= \alpha_0 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{aligned}$$

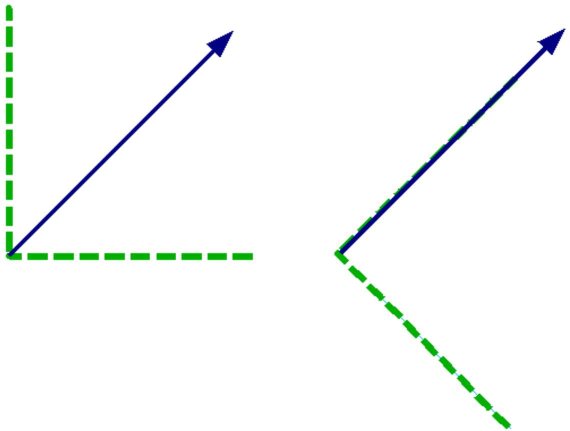
What is a qutrit?

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle$$
$$= \alpha_0 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \alpha_1 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$P|0\rangle = |\alpha_0|^2$$
$$P|1\rangle = |\alpha_1|^2$$
$$P|2\rangle = |\alpha_2|^2$$

Measurements

Measuring causes superpositions to *collapse*. Given orthogonal basis, system must "choose" state.



Entanglement *or* Spooky Action at a Distance

Entanglement is quintessential quantum mechanics. EPR shows that it is *non-local*.

$$\begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \\ \gamma_5 \\ \gamma_6 \\ \gamma_7 \\ \gamma_8 \\ \gamma_9 \end{pmatrix} \neq \begin{pmatrix} \alpha_1\beta_1 \\ \alpha_1\beta_2 \\ \alpha_1\beta_3 \\ \alpha_2\beta_1 \\ \alpha_2\beta_2 \\ \alpha_2\beta_3 \\ \alpha_3\beta_1 \\ \alpha_3\beta_2 \\ \alpha_3\beta_3 \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \otimes \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$$

Entanglement *or* Spooky Action at a Distance

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

Entanglement *or* Spooky Action at a Distance

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

Entanglement *or* Spooky Action at a Distance

$$\begin{aligned}\frac{|01\rangle + |10\rangle}{\sqrt{2}} &= \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle \\ &= 0|00\rangle + \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + 0|11\rangle\end{aligned}$$

Entanglement *or* Spooky Action at a Distance

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$$= 0|00\rangle + \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + 0|11\rangle$$

$$P|0\rangle = \frac{1}{2}$$

Entanglement *or* Spooky Action at a Distance

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$$= 0|00\rangle + \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + 0|11\rangle$$

$$P|0\rangle = \frac{1}{2} \quad \longrightarrow \quad P|0\rangle = 0,$$

Entanglement *or* Spooky Action at a Distance

$$\begin{aligned}\frac{|01\rangle + |10\rangle}{\sqrt{2}} &= \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle \\ &= 0|00\rangle + \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + 0|11\rangle\end{aligned}$$

$$P|0\rangle = \frac{1}{2} \quad \longrightarrow \quad \begin{aligned} P|0\rangle &= 0, \\ P|1\rangle &= 1 \end{aligned}$$

Entanglement *or* Spooky Action at a Distance

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$
$$= 0|00\rangle + \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + 0|11\rangle$$

$$P|0\rangle = \frac{1}{2} \quad \longrightarrow \quad P|0\rangle = 0,$$
$$P|1\rangle = 1$$
$$P|1\rangle = \frac{1}{2}$$

Entanglement *or* Spooky Action at a Distance

$$\begin{aligned}\frac{|01\rangle + |10\rangle}{\sqrt{2}} &= \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle \\ &= 0|00\rangle + \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + 0|11\rangle\end{aligned}$$

$$\begin{aligned}P|0\rangle = \frac{1}{2} \quad &\longrightarrow \quad P|0\rangle = 0, \\ &\qquad\;\; P|1\rangle = 1 \\ P|1\rangle = \frac{1}{2} \quad &\longrightarrow \quad P|0\rangle = 1,\end{aligned}$$

Entanglement *or* Spooky Action at a Distance

$$\frac{|01\rangle + |10\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$
$$= 0|00\rangle + \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle + 0|11\rangle$$

$$P|0\rangle = \frac{1}{2} \longrightarrow P|0\rangle = 0, \quad P|1\rangle = 1$$

$$P|1\rangle = \frac{1}{2} \longrightarrow P|0\rangle = 1, \quad P|1\rangle = 0$$

Work

Product Measurements

- For state $|\psi\rangle$ in $\mathbb{C}^3$, we choose a basis.

$$\{|a_1\rangle, |a_2\rangle, |a_3\rangle\}$$

Product Measurements

- For state $|\psi\rangle$ in $\mathbb{C}^3$, we choose a basis.

$$\{|a_1\rangle, |a_2\rangle, |a_3\rangle\}$$

- For state $|\varphi\rangle$ in $\mathbb{C}^3 \otimes \mathbb{C}^3$, we choose two bases.

$$\{|a_1\rangle, |a_2\rangle, |a_3\rangle\}$$
$$\{|b_1\rangle, |b_2\rangle, |b_3\rangle\}$$

Product Measurements

- For state $|\psi\rangle$ in $\mathbb{C}^3$, we choose a basis.

$$\{|a_1\rangle, |a_2\rangle, |a_3\rangle\}$$

- For state $|\varphi\rangle$ in $\mathbb{C}^3 \otimes \mathbb{C}^3$, we choose two bases.

$$\{|a_1\rangle, |a_2\rangle, |a_3\rangle\}$$
$$\{|b_1\rangle, |b_2\rangle, |b_3\rangle\}$$

For a product measurement on $|\varphi\rangle$, we choose one basis for the first system, then second basis for second system.

Entangled Measurements

Project on nine *entangled* orthonormal vectors in $\mathbb{C}^3 \otimes \mathbb{C}^3$ as basis.

$$
\begin{aligned}
&|e_1\rangle, \quad |e_2\rangle, \quad |e_3\rangle, \\
&|e_4\rangle, \quad |e_5\rangle, \quad |e_6\rangle, \\
&|e_7\rangle, \quad |e_8\rangle, \quad |e_9\rangle
\end{aligned}
$$

Question

Given 3 orthonormal states in $\mathbb{C}^3 \otimes \mathbb{C}^3$, can we distinguish with certainty using a product measurement?

Answer

Conjecture

Yes!

Answer

Conjecture

Yes!

How?

Answer

Conjecture

Yes!

How?

- First, measure first system. . .

Answer

Conjecture

Yes!

How?

- First, measure first system. . .
- . . . then use outcome to choose basis for second sytem.

Answer

Conjecture

Yes!

How?

- First, measure first system. . .
- . . . then use outcome to choose basis for second sytem.

We show it is also true for $\mathbb{C}^3 \otimes \mathbb{C}^n$, if we measurement the first system *first*.

Optimization as Technique

- We use TOMLAB package to find global minima for function describing how orthogonal three vectors are.

Optimization as Technique

- We use TOMLAB package to find global minima for function describing how orthogonal three vectors are.
- We search over rotations of these vectors.

Open Problems

- Analytical proof of conjecture

Open Problems

- Analytical proof of conjecture
- "Ping-pong" protocol for higher dimensions

Further Reading

- **Reliably distinguishing states in qutrit channels using one-way LOCC**
  Christopher King and Daniel Matysiak
  Preprint: `http://arxiv.org/abs/quant-ph/`**0510004**
  Submitted to *Quantum Information and Computation*